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# Dynamic Forces in Spur Gears—Measurement, Prediction, and Code Validation

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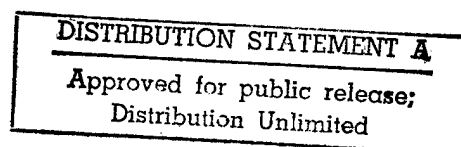
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# **DYNAMIC FORCES IN SPUR GEARS -- MEASUREMENT, PREDICTION, AND CODE VALIDATION**

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## **ABSTRACT**

Measured and computed values for dynamic loads in spur gears were compared to validate a new version of the NASA gear dynamics code DANST-PC. Strain gage data from six gear sets with different tooth profiles were processed to determine the dynamic forces acting between the gear teeth. Results demonstrate that the analysis code successfully simulates the dynamic behavior of the gears. Differences between analysis and experiment were less than 10 percent under most conditions.

## **INTRODUCTION**

The gearbox is a major source of helicopter cabin noise which may exceed 100 decibels sound pressure level. NASA and the US Army have sponsored research projects to find ways to reduce this noise.

Noise excitation in a transmission is caused by the load fluctuation as gear teeth enter and leave mesh. The cyclic variation in the numbers of teeth carrying the load causes a periodic change in the gear mesh stiffness and affects the relative position of the gears. Any deviation in the angular position of the driven gear from its ideal position is considered to be transmission error. Transmission error arises from manufacturing and mounting errors as well as tooth deflection under load.

Gear designs often include modified tooth profiles (tip relief) to minimize transmission error. Computer codes allow gear designers to investigate the effect of profile modifications on transmission error

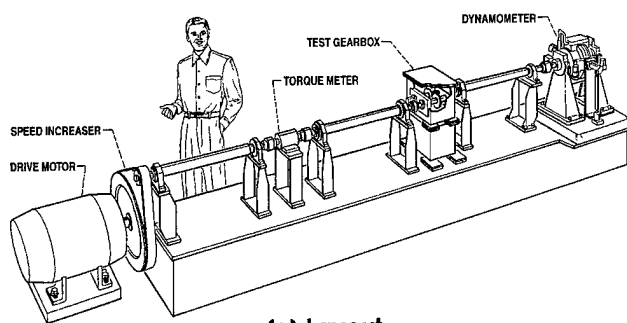
and gear dynamics. Reported studies of spur gear profile modification include Tavakoli and Houser (1984), Munro, et al. (1990), and Lin et al. (1989). Earlier dynamic strain gage experiments were reported in Rebbechi, et al. (1991), and Oswald, et al. (1991). The strain data used here were previously presented in Oswald and Townsend (1995).

The goal for the research reported in this paper was to determine the dynamic tooth forces (loads) from dynamic strain measurements and to compare the results with predictions of a new version of the NASA gear dynamics code now called DANST-PC. Data presented here include time domain strain measurements, the corresponding dynamic loads for the test gears and predictions of the dynamic loads.

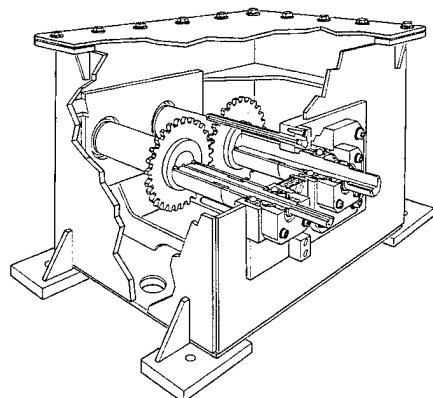
## **APPARATUS**

Tests were performed on the NASA gear noise rig (figure 1). The rig features a single-mesh gearbox powered by a 150 kW (200hp) variable speed electric motor. An eddy-current dynamometer loads the output shaft. The gearbox can operate at speeds up to 6000 rpm. The rig was built to carry out fundamental studies of gear noise and the dynamic behavior of gear systems. It was designed to allow testing of various configurations of gears, bearings, dampers and supports. The gearbox is extensively instrumented for strain, noise and vibration measurements.

A poly-V belt drive was used as a speed increaser between the 1750 rpm motor and the input shaft. A soft coupling on the input shaft reduces input torque fluctuations caused by the belt splice



(a) Layout



(b) Detail of gearbox

Figure 1.--NASA gear noise rig

The gearbox oil inlet temperature was maintained at  $70 \pm 2^\circ$  Celsius for these tests. At the mean temperature of  $70^\circ$ , the viscosity of the synthetic turbine engine oil (MIL-L-23699B) used in the tests is 9.5 centistoke.

#### Test Gears:

The test gears were identical spur gears (at 1:1 ratio) machined to master gear (AGMA Class 15) accuracy. Test gear parameters are shown in Table 1. Profile modifications were chosen to compensate for tooth deflection under load. No additional allowance was made for manufacturing errors since these errors are less than one-tenth of the computed deflection at the nominal load of 71.8 N-m (635 lb-in).

We tested six different gear profiles. These include an unmodified profile, and combinations of linear and parabolic profile modification (tip relief). Additional data on the gear profiles is given in Oswald and Townsend (1995).

#### Instrumentation:

General-purpose, constantan foil, resistance strain gages with gage length 0.38 mm (0.015 in) were installed in the tooth-root fillets

TABLE 1, Test Gear and Rig Modeling Parameters

|   |                               |
|---|-------------------------------|
| Gear Tooth Type   | Standard, full-depth          |
| No. teeth   | 28 and 28                     |
| Module, mm (diametral pitch, $\text{in}^{-1}$ )                     | 3.175 (8)                     |
| Face width, mm (in)   | 6.35 (0.25)                   |
| Pressure Angle, deg   | 20                            |
| Theoretical contact ratio   | 1.64                          |
| Tooth root radius, mm (in)  | 1.35 (0.053)                  |
| Max. tooth spacing error, $\mu\text{m}$ (in)                        | 0.18 (0.00007)                |
| Max. profile error, $\mu\text{m}$ (in)                              | 0.13 (0.00005)                |
| Mesh damping coeff.   | 10% of critical               |
| Gear inertia, $\text{kg}\cdot\text{m}^2$ (lb-in-sec <sup>2</sup> )  | $67 \times 10^{-6}$ (0.00594) |
| Motor inertia, $\text{kg}\cdot\text{m}^2$ (lb-in-sec <sup>2</sup> ) | 0.0011 (0.100)                |
| Load inertia, $\text{kg}\cdot\text{m}^2$ (lb-in-sec <sup>2</sup> )  | 0.0014 (0.124)                |
| Input stiffness, N-m/rad (lb-in/rad)                                | 17,000 (150,000)              |
| Output stiffness, N-m/rad (lb-in/rad)                               | 17,000 (150,000)              |

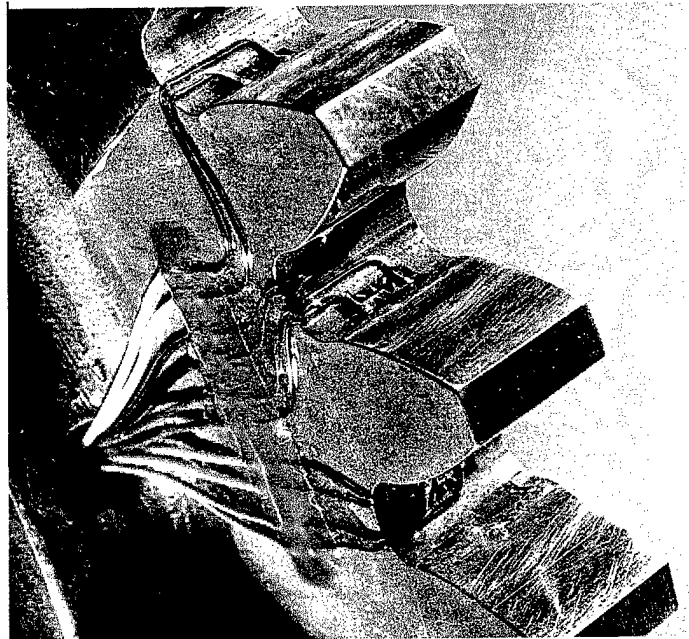
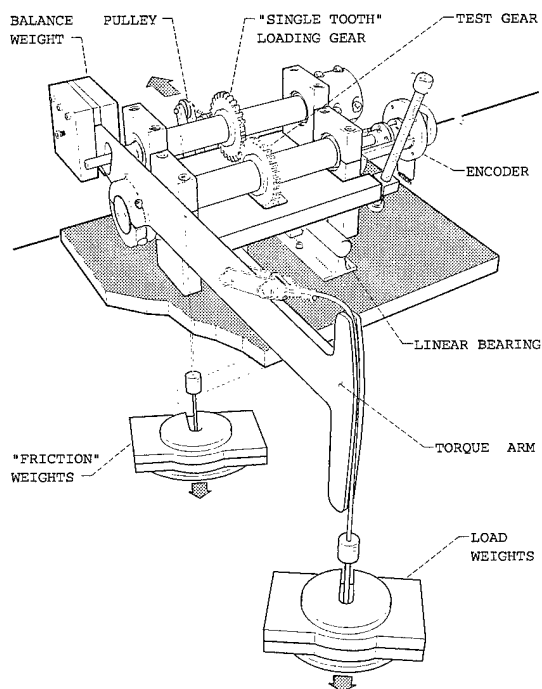


Figure 2.--Strain gage installation on test gear

on both the loaded (tension) and unloaded (compression) side of two adjacent teeth on the output (driven) gear (Fig. 2). To measure maximum tooth bending stress, the gages were placed at the  $30^\circ$  tangency location (Cornell, 1980). Two methods of signal conditioning were used on strain gage signals. For static measurement, a strain gage (Wheatstone) bridge was used. For dynamic measurements, the strain gages were connected through a slip-ring assembly to constant-current strain gage amplifiers.

A 12-bit digital data acquisition system was used to record strain data. The sample rate was varied from 6.6-50 kHz per channel to



**Figure 3.--Gear strain gage calibration rig**

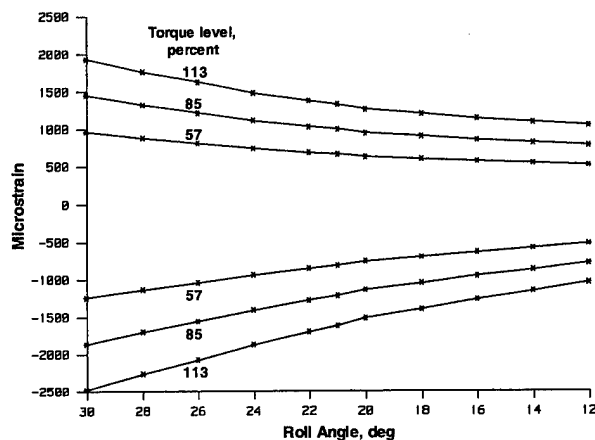
provide 500 samples per revolution for each channel. A once-per-rev pulse from a transducer on the input gear shaft provided a timing signal. The transducer was adjusted so the leading edge of the pulse occurred at a known roll angle for a reference tooth on the gear. This allows us to determine the roll angle for any point in the data record.

## TEST PROCEDURE

### Static Calibration Data:

We recorded static (non-rotating) strain using a special calibration rig (Fig. 3) to construct a matrix of tooth force influence coefficients (Rebbechi, et al., 1991). These coefficients relate measured strains to the normal and frictional forces acting between gear teeth. The rig allows separate control over normal and frictional forces on the gear tooth. One gear-shaft assembly is mounted on linear bearings such that any unbalanced traction force on the tooth surface will cause motion of the shaft assembly. The instrumented gear meshes with a special calibration gear that has several teeth ground away so that loading is applied to only a single tooth. A weight and pulley system applies a controlled load to the system which is resisted by tooth friction. The roll angle is measured by a 14-bit absolute encoder.

First, we performed "frictionless" calibration with no load applied via the "friction" weight pan. We recorded static strains at two degree increments for roll angles from 32 to 10 degrees. An extra



**Figure 4.--Single-tooth frictionless calibration data**

value was recorded at 21 degrees since this is close to the pitch point (20.85°) on the gears. (Note: the strain gages are on the driven gear where contact starts near the tip and proceeds towards smaller roll angles at the root of the tooth.) For each test position, readings were taken with zero friction and at torque levels of 57, 85 and 113 percent of the nominal torque of 71.8 N-m (635 lb-in). Figure 4 shows a sample of the frictionless calibration data. It includes both tensile and compressive strains for one tooth at the three load levels.

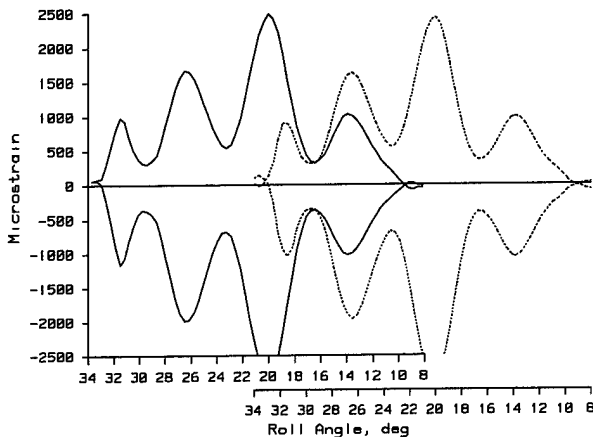
After frictionless calibration, the process was repeated "with friction". The highest torque (113 percent) was applied and strains were recorded at friction loads of 0, 100, and 190 Newtons. (The 190 N friction load was the highest that could typically be carried without slip.)

Calibration was performed separately for each of the two instrumented teeth on the test gear. Three trials of all measurements were averaged together to reduce errors.

The calibration data were used to generate a tooth force influence coefficient matrix. This matrix relates normal and tangential (frictional) forces between a pair of teeth to the strain readings on the two sides of one tooth. Sixth-degree polynomials were computed from the influence coefficients. These polynomials allow interpolation of forces (both normal and frictional forces) for any roll angle within the zone of tooth contact. The matrix procedure is described in the appendix of Rebbechi, et al. (1991).

### Dynamic Strain Data:

Dynamic strains were recorded for the six gear pairs at 36 torque-speed test conditions: 9 torque levels (16, 31, 47, 63, 79, 110, 126, and 142 percent of the nominal torque of 71.8 N-m (635 lb-in)) and 4 speeds (800, 2000, 4000, 6000 rpm).



**Figure 5.--Typical dynamic strain data (gear set A, 4000 rpm, 142 percent torque)**

Data from each sample were digitally resampled, using linear interpolation, at 1000 samples per revolution and synchronously averaged. Time domain synchronous averaging is in wide use for gear diagnostics (McFadden, 1987). It was used here to reduce random "noise" effects (such as torque fluctuation caused by the belt drive). Its implementation requires at least two channels of data -- a timing signal plus the data of interest. The timing signal provided the resample intervals needed for exactly one revolution of the gear. We repeated each test condition three times. The data files from three trials were plotted over each other (to check for consistent results) then averaged together. Figure 5 shows a sample of dynamic strain data measured on the loaded (tension) and unloaded (compression) sides of two adjacent teeth. Averaged strain data were used to calculate dynamic gear tooth forces.

## ANALYTICAL PROCEDURE

We used DANST-PC, a new version of the NASA gear dynamics code DANST, to model the dynamic loads (tooth contact forces) of the test gears. (The original DANST is described in Oswald, et al., 1993.) A new feature in DANST-PC accounts for the increased length of tooth contact due to tooth flexibility for gears operating under high loads (Lin et al., 1993).

DANST employs 4 degrees of freedom to represent the torsional response of input (motor), the two gears and output (load). Equivalent mass (inertia) and stiffness elements represent the input and output of the gear noise rig.

As we compared analytical and experimental results, we found better correlation if the inertia of the gears was increased to include the inertia of the gear shafts. Because the gears connect to the shafts through press-fits and keys, it is reasonable to consider the gears and

shafts as a single element. The values used for the inertia and stiffness are shown in Table 1.

DANST-PC can model gears with involute or modified tooth profiles. Profile modification is in the form of tip relief, where material was removed from the tooth tip to compensate for tooth deflection under load. The program can internally generate four "standard" types of tip relief and it can also accept "digitized" profile inputs. The DANST modification schemes are defined in Oswald, et al., 1993. The profiles of the six gear sets tested in this research are shown in Oswald and Townsend, (1995). Where the gear profiles differed significantly from the standard profile types, the actual profiles were entered digitally.

## RESULTS AND DISCUSSION

To compare experimental and analytical results, we plotted the measured dynamic loads next to the predicted dynamic loads. (The dynamic loads are the normal forces between gear teeth.) Figures 6 and 7 show the measured and predicted dynamic loads for two different gear sets. Each figure includes separate plots at the four speeds with five different torque levels on each plot. (We took data at nine torque levels but we show only five here to improve visibility.)

In Fig. 6, we compare measured and predicted dynamic loads for gear set "A". These gears have an involute profile with no relief. As one would expect, set A gears show fairly smooth response at low torque and a much more dynamic response at higher torque. At 4000 rpm, dynamic loads are very high. These occur at twice tooth meshing frequency.

The predicted and measured values match each other rather well in Fig. 6, both in the general pattern and in magnitude, especially at 2000 and 4000 rpm. There are some differences in the waveforms at 800 and 6000 rpm. Some of the differences may be due to external "blending" effects not considered in the model such as load fluctuations from the motor and belt drive or low-frequency vibration modes of the long shafts connecting the motor and dynamometer to the gearbox.

In Fig. 7, we compare measured and predicted dynamic loads for gear set "D". These gears have linear tip relief that extends about 88 percent of the distance from the tip to the high point of single tooth contact. This length of relief falls in the class Munro, et al. (1990) designates as "intermediate" relief. He recommends intermediate relief for gears that operate over a range of torque levels.

In both the measured and predicted dynamic load curves, set D gears show fairly smooth dynamic response except at light torque,

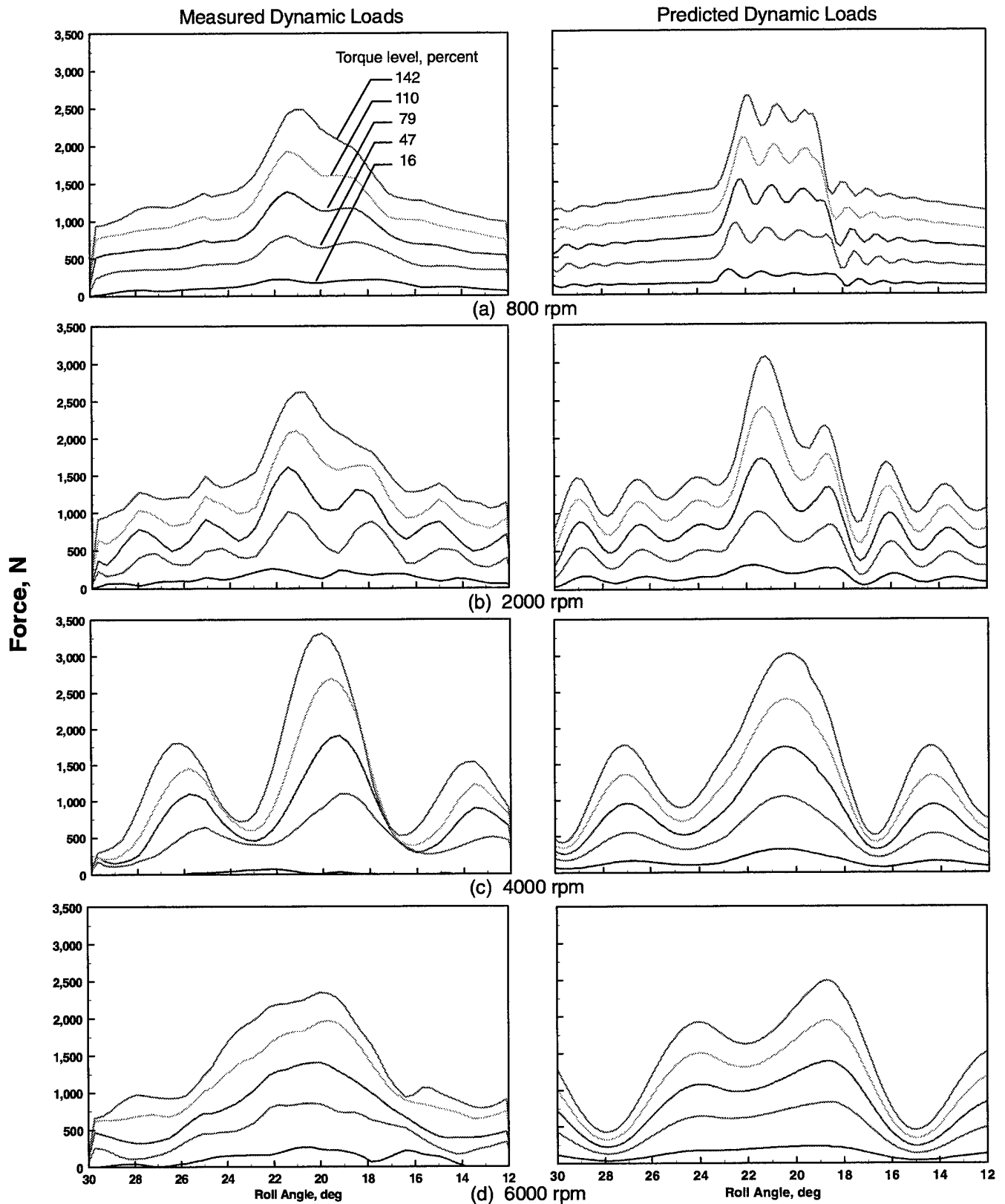
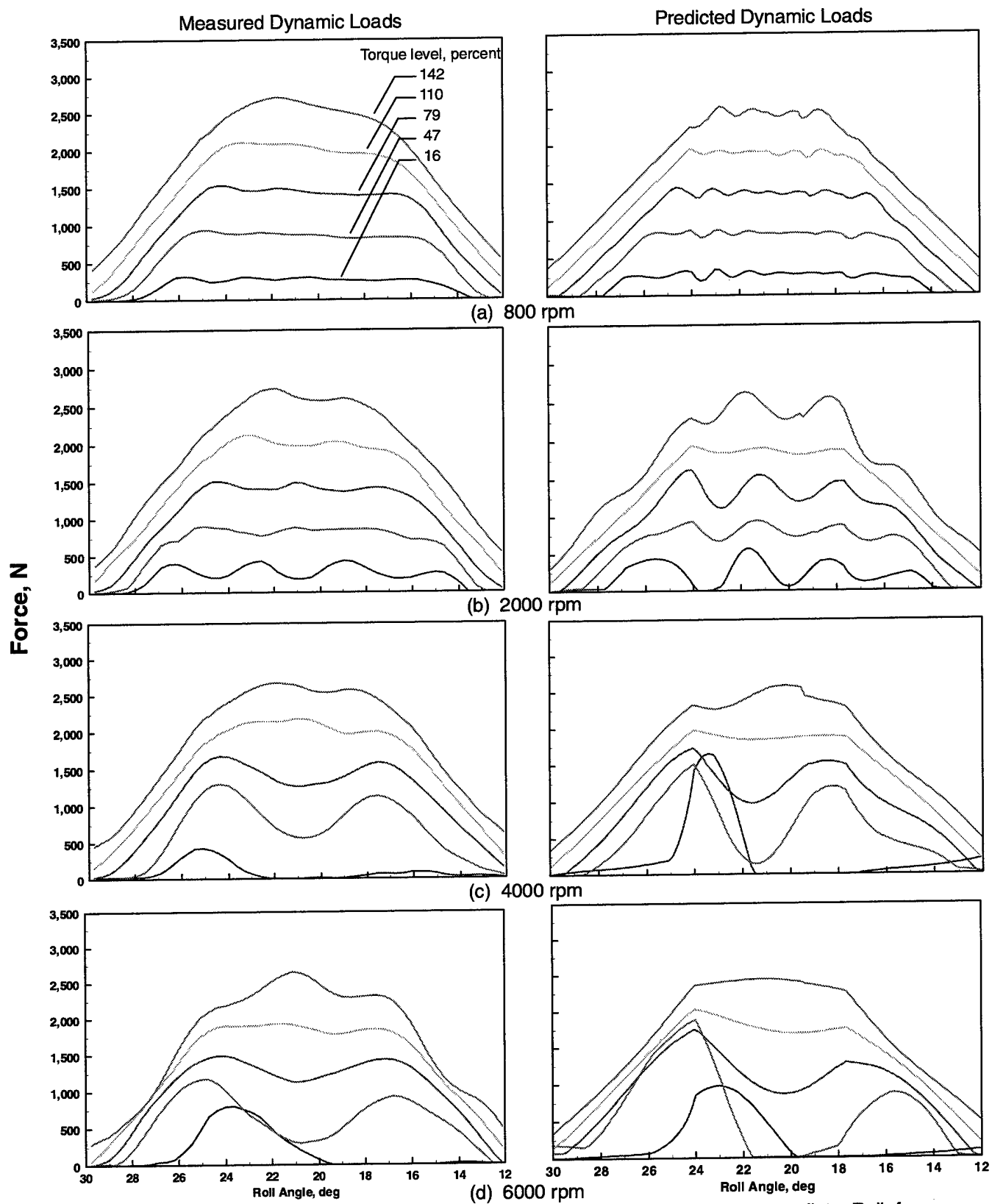
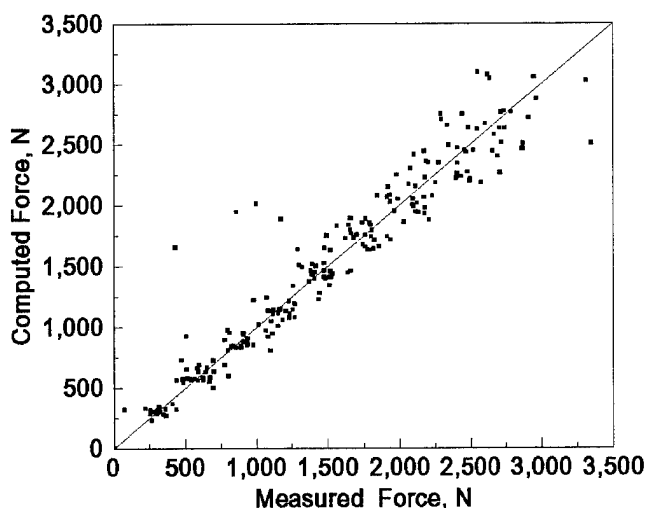


Figure 6-- Measured and Predicted Dynamic Loads, Test Gear Set A, No Relief





**Figure 8.--Comparison of computed and measured maximum dynamic tooth force**

high speed conditions. At the two highest speeds (4000 and 6000 rpm) and the lowest torque (16 percent) there is a very short and strong dynamic load spike followed by the teeth "bouncing" out of contact.

To summarize the correlation between the dynamic loads predicted by DANST-PC and those measured by our strain-gage technique, we plotted (Figure 8) the maximum dynamic load predicted by DANST versus the maximum measured values for the six gear sets tested and for all 36 test conditions (216 tests). The diagonal line in the figure shows where measured and experimental values agree. Most data falls within a band straddling the line with an error of less than ten percent. A few points are far from the line. These are generally from high-speed, low-torque conditions where the analysis overestimated the dynamic load. Two of the points with the worst agreement are from the lowest torque curves for gear set D shown in Fig. 7 (c) and 7 (d).

## SUMMARY AND CONCLUSIONS

Low contact ratio spur gears with six different profiles were tested in the NASA gear noise rig. Dynamic tooth bending strains were recorded for each gear design at 36 operating conditions. The strains were converted to dynamic force data using static calibration data collected on a special calibration rig. The experimental results were compared to analytical data from the gear dynamics code DANST-PC. The following conclusions were drawn from the data:

(1) The predicted (computed) and experimental results generally agreed both in magnitude and in the shape of the curves.

(2) The predicted value for the peak dynamic force agreed with the experimental values within about ten percent except for the high-speed, low torque cases where the analysis overestimated the dynamic effect.

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